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Spontaneous reduction of noncommutative gauge symmetry and model building

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Abstract. We propose a mechanism for the spontaneous (gauge-invariant) reduction of noncommutative $\mathcal{U}(n)$ gauge theories down to $SU(n)$. This can be achieved through the condensation of composite $\mathcal{U}(n)$ gauge invariant fields that involve half-infinite Wilson lines in the trace-$U(1)$ noninvariant and $SU(n)$ preserving direction. Based on this mechanism, we discuss an anomaly-free fully gauge invariant noncommutative standard model based on the minimal gauge group $\mathcal{U}(3)\times\mathcal{U}(2)\times\mathcal{U}(1)$, previously proposed, and show how it can be consistently reduced to the standard model with the usual particle spectrum. Charge quantization for quarks and leptons naturally follows from the model.

1 Introduction

Noncommutative (NC) space naturally emerges in string theory in the presence of the nonzero background B-field (see, e.g. the reviews [1–3] and references therein). If we seriously accept this possibility, an important task is to 'reproduce' the known physics at low energies, which is described by the celebrated standard model with an amazing accuracy. The construction of a consistent noncommutative standard model (NCSM), however, faces significant difficulties. One is related with restrictions imposed by noncommutative group theory and gauge invariance. Namely: (i) only $\mathcal{U}(n)$[1] unitary gauge theories (as well as direct products of different $\mathcal{U}(n_i)'s$, $\prod_{i=1}^{k}\times\mathcal{U}(n_i)$) admit noncommutative extension [4][2], but not $SU(n)'s$; (ii) nontrivial representations of noncommutative $\mathcal{U}(n)$ are constrained to be fundamental (left module), antifundamental (right module), or adjoint (left-right module) only. In addition, the only allowed nontrivial representations of the product of gauge groups $\prod_{i=1}^{k}\times\mathcal{U}(n_i)$ are those transforming as fundamental–antifundamental under two $\mathcal{U}(n_i)$ factors at most [5–7].

An interesting way of circumventing these group-theoretical problems is through an alternative approach to NC gauge theories based on the expansion in the NC parameter and Seiberg–Witten map. This approach admits NC $SU(n)$ gauge theories [8]. The model building with this alternative approach can be found, e.g. in [9, 10].

However, just from the above group-theoretic properties it is evident that straightforward (based on the Weyl–Moyal approach) noncommutative extension of the standard model gauge group (that is, $G_{\mathrm{NCSM}}=\mathcal{U}(3)\times\mathcal{U}(2)\times\mathcal{U}(1)$) already contains new particles – two extra gauge bosons (the rank of G_{NCSM} is 6 vs. 4 of $G_{\mathrm{SM}}=SU(3)\times SU(2)\times U(1)$). Moreover, there is the problem of matter (quark-lepton) representations as well. Namely, since the only allowed charges within the noncommutative $\mathcal{U}(1)$ are $0, \pm 1$ [11], it is clear that $\mathcal{U}(1)$ cannot be identified with the usual weak hypercharge to account for the fractional electric charges of the quarks. Hence a different embedding of the electric charge in G_{NCSM} must be found.

An attempt to solve these problems has been made in [7]. The extra gauge bosons are made massive, leaving just the SM gauge group G_{SM} at low energies. This was achieved by introduction of the so-called Higgsac fields, which transform under the trace-$U(1)$ parts of G_{NCSM}. The matter content has been chosen exactly as in the usual standard model, but now obeying the no-go theorem [6][3].

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[1] Calligraphic letters denote noncommutative gauge groups (e.g. $\mathcal{U}(n)$), while italic letters denote commutative groups (e.g. $SU(n)$).

[2] Recall that in the noncommutative case $\mathcal{U}(n)\neq SU(n)\times U(1)$, while $U(n)=SU(n)\times U(1)$ in the commutative case.

[3] As far as the particle classification is concerned, the use of the representations of the usual Poincaré symmetry has been recently justified, when it was noticed that noncommutative field theories with commutation relation $[x_\mu, x_\nu]=i\theta_{\mu\nu}$, with $\theta_{\mu\nu}$ an antisymmetric constant matrix, are invariant under twisted Poincaré algebra [12], whose representations are the same as those of the usual Poincaré group.

Remarkably, upon the Higgsac condensation, a linear combination of trace-$U(1)'s$ in G_{NCSM}, which remains massless, is just the weak hypercharge and thus the fractional charges of quarks are explained automatically. This is a very welcome outcome of the model and somehow reminds us of the charge quantization within the usual commutative models of grand unification[4].

Unfortunately, the above nice picture has a serious theoretical drawback. The point is that the symmetry breaking by the Higgsac field is not spontaneous. As a result, one accounts for the violation of unitarity in gauge boson scattering at high energies [14]. Another problem of the model of [7] is that it contains gauge anomalies related with an extra trace-$U(1)'s$ in G_{NCSM}. As usual, one can add extra matter fields to make each G_{NCSM} factor vector-like and hence make the whole theory anomaly-free. Upon the symmetry breaking down to G_{SM}, this extra matter is vector-like, and, in principle, can pick up mass through the Yukawa couplings with the appropriate Higgsac fields. However, once again these Yukawa couplings are not G_{NCSM} gauge invariant. Summarizing the above discussion, it seems that the problems of the model of [7] can be avoided by finding a proper gauge invariant realization of the Higgsac mechanism. The scope of this paper is to propose such a mechanism, by using the construction of noncommutative generalizations of the gauge invariant local operators, based on Wilson lines [15] (see also [16, 17]).

2 Spontaneous NC gauge symmetry breaking

Consider the ‘canonical’ NC space-time defined through the $*$-commutation relations,

$$[x_\mu, x_\nu]_* = i\theta_{\mu\nu} \;, \tag{1}$$

where $\theta_{\mu\nu}$ is an antisymmetric constant matrix. The x_μ in (1) are the ordinary 4-coordinates with $*$-multiplication defined as:

$$f(x) * g(x) = \exp\left(\frac{i}{2}\theta_{\mu\nu}\partial_{x_\mu}\partial_{y_\nu}\right) f(x)g(y)\bigg|_{x=y} .$$

On this NC space-time we define NC gauge theory based on the gauge group $\mathcal{U}(n)$. The n^2 gauge bosons form an adjoint representation of $\mathcal{U}(n)$:

$$A^\mu_X(x) \longrightarrow u(x) * \left(A_\mu(x) - \frac{i}{g}\mathbf{1}_{n\times n}\partial_\mu\right) * u^{-1}(x) \,, \tag{2}$$

where $u(x) = \exp_*(-ig\beta^A(x)T^A)$ is an element (defining representation) of the $\mathcal{U}(n)$ group, $A_\mu(x) = A^A_\mu(x)T^A$ is an $u(n)$-algebra valued gauge field with generators $T^A = \frac{1}{2}\lambda^A$, where $\lambda^1, \ldots, \lambda^{n^2-1}$ are the generalized Gell–Mann matrices and $T^0 = \mathbf{1}_{n\times n}$, and g is the gauge coupling constant.

Recall that commutative $U(n)$ gauge symmetry is broken spontaneously down to the $SU(n)$ subgroup once a $SU(n)$-singlet and $U(1)$-charged scalar field acquires nonzero vacuum expectation value. One of such allowed (in commutative case) representations is n-index totally antisymmetric tensor representation[5]

$$\phi^{[i_1 i_2 \ldots i_n]}(x) \,, \tag{3}$$

out of which the scalar field $\phi(x)$ can be constructed in the form

$$\phi(x) = \frac{1}{n!}\epsilon_{i_1 i_2 \ldots i_n}\phi^{[i_1 i_2 \ldots i_n]}(x) \,. \tag{4}$$

The field $\phi(x)$ in (4) carries the $U(1)$ charge equal to n and is the representation of the Higgsac field used in [7]. However, the noncommutative $\mathcal{U}(n)$-transformations do not close when acting on the Higgsac field Φ, and hence the field Φ is not a representation of the $\mathcal{U}(n)$ group. Subsequently, the symmetry breaking in [7] is not spontaneous, since it goes through a gauge noninvariant mechanism.

However, according to the prescription of [15, 16], one can construct operators that are invariant under a noncommutative gauge group ($\mathcal{U}(n)$ in this case), out of operators that are gauge invariant under the corresponding commutative group (i.e. $U(n)$), but no longer invariant under the noncommutative gauge group, as is the case of $\phi(x)$ defined in (4). Consequently, to restore the gauge invariance, we construct the Higgsac field by introducing, instead of (4), the following scalar field (the gauge-invariant Higgsac field):

$$\Phi(x) = \frac{1}{n!}\epsilon_{i_1 i_2 \ldots i_n} W^{i_i}_{j_1} * W^{i_2}_{j_2} * \ldots * W^{i_n}_{j_n} * \phi^{[j_1 j_2 \ldots j_n]}(x) \,, \tag{5}$$

where

$$\begin{aligned} W &= P_* \exp\left(ig\int_0^1 \mathrm{d}\sigma \frac{\mathrm{d}\xi^\mu}{\mathrm{d}\sigma} A_\mu(x+\xi(\sigma))\right) \\ &= \mathbf{1}_{n\times n} + \sum_{n=1}^{\infty}\frac{(ig)^n}{n!}\int_0^1 \mathrm{d}\sigma_1 \int_{\sigma_1}^1 \mathrm{d}\sigma_2 \ldots \int_{\sigma_{n-1}}^1 \mathrm{d}\sigma_n \\ &\quad \times \frac{\partial\xi^{\mu_1}}{\partial\sigma_1} \ldots \frac{\partial\xi^{\mu_n}}{\partial\sigma_n} A_{\mu_1}(x+\xi(\sigma_1)) * \ldots * A_{\mu_n}(x+\xi(\sigma_n)) \end{aligned} \tag{6}$$

is a half-infinite Wilson line. With path ordering defined with respect to the $*$-product, the contour C is:

$$C = \{\,\xi^\mu(\sigma), \quad 0 < \sigma < 1 \,|\, \xi^\mu(0) = \infty, \quad \xi^\mu(1) = 0\} \,,$$

[4] Another approach to the charge quantization problem is to find a different embedding of the electric charge in an extended noncommutative gauge symmetry. This has been recently discussed in [13] within a model with $\mathcal{U}(4)\times\mathcal{U}(3)\times\mathcal{U}(2)$ gauge symmetry, where several conditions are fulfilled. Besides the extended gauge group, however, the model requires the introduction of three extra generations of mirror quarks and leptons in order to achieve anomaly cancellation.

[5] Notice that, due to the constraints on the representations of NC groups [5, 6] (i.e. matter fields can be only in fundamental, antifundamental, adjoint or singlet representations of NC $\mathcal{U}(n)$), the auxiliary tensor representation in (3) is not a representation of the NC $\mathcal{U}(n)$ gauge group.

and $\phi^{[j_1 j_2,\cdots,j_n]}(x)$ is an antisymmetric n-index object under $\mathcal{U}(n)$. The actual shape of the Wilson line (6) is not important and thus it can be arbitrary. Within the physically admissible gauge transformations (i.e. those for which $u(x) \to \mathbf{1}$ when $x \to \infty$) this Wilson line transforms as an antifundamental object

$$W(x) \to W^u(x) = W(x) * u^{-1}(x)\,. \tag{7}$$

Then the composite Higgsac field Φ in (5) is a gauge-invariant object [16, 17],

$$\begin{aligned}\Phi^u(x) &= \frac{1}{n!}\epsilon_{i_1 i_2 \ldots i_n}\left[(W^{i_1} \otimes W^{i_2} \otimes \ldots \otimes W^{i_n}) * \phi\right]^u \\ &= \frac{1}{n!}\epsilon_{i_1 i_2 \ldots i_n}\left[(W^u * u)^{i_1} \otimes (W^u * u)^{i_2}\right. \\ &\qquad \left.\otimes \ldots \otimes (W^u * u)^{i_n}\right] * \phi = \Phi(x)\,.\end{aligned} \tag{8}$$

Using the Taylor expansion (6) of the Wilson lines in (5),

$$\Phi(x) = \phi(x) + \ldots\,,$$

we see that the first term in the expansion is just the ordinary Higgsac field (4), while the rest of the terms provide a gauge invariant completion. Now, if the field $\Phi(x)$ develops a nonzero vacuum expectation value along the Higgsac direction, i.e.

$$\langle \Phi(x) \rangle = \langle \phi(x) \rangle = \text{const.}\,,$$

the NC $\mathcal{U}(n)$ gauge symmetry becomes spontaneously broken down to $SU(n)$. Indeed, since $\Phi(x)$ is the gauge-singlet field we can write a simple Lagrangian for it:

$$\mathcal{L}_{\text{Higgsac}} = \partial_\mu \Phi \partial^\mu \Phi^* - V(\Phi\Phi^*)\,, \tag{9}$$

where

$$V(\Phi\Phi^*) = m^2 \Phi\Phi^* + \frac{\lambda}{2}(\Phi * \Phi^*)^2 \tag{10}$$

is the bounded from below ($\lambda > 0$) tachyonic potential ($m^2 < 0$). The Lagrangian (9) can be viewed as a gauge-invariant version of the Higgsac Lagrangian proposed in [7]. As in the ordinary commutative case, we assume that the perturbative vacuum for the gauge field is given by the pure gauge configuration equivalent to the trivial vector potential, i.e. $\langle A_\mu \rangle = 0$. Then $\langle W \rangle = \mathbf{1}_{n\times n}$, and the potential (10) is reduced to the potential for the Higgsac field $\phi(x)$, $V(\phi\phi^*)$, with a nontrivial minimum that can be chosen as:

$$\langle \phi(x) \rangle = \sqrt{-\frac{m^2}{\lambda}}\,. \tag{11}$$

Hence, we expect that the trace-$U(1)$ field of NC $\mathcal{U}(n)$ gauge theory picks up a mass leaving $SU(n)$ unbroken. To see this, we must closely inspect the kinetic term in (9). First note that the leading order term (θ-independent) in θ-expansion for the composite object (5) looks like

$$\begin{aligned}\Phi(x) &= (\det W)\,\phi(x) \\ &= \left(1 + ig\int_0^1 \mathrm{d}\sigma \frac{\mathrm{d}\xi^\mu}{\mathrm{d}\sigma} Tr A_\mu(x + \xi(\sigma)) + \ldots\right)\phi(x)\,.\end{aligned}$$

Hence, the expansion of $\partial_\mu \Phi(x)$ contains the ordinary covariant derivative for the Higgsac field, i.e.

$$\begin{aligned}\partial_\mu \Phi(x) &= \left(\partial_\mu + ingA_\mu^0\right)\phi(x) + ig \\ &\quad \times \left[\int_0^1 \mathrm{d}\sigma \frac{\mathrm{d}\xi^\mu}{\mathrm{d}\sigma} Tr A_\mu(x + \xi(\sigma))\right]\partial_\mu \phi(x) + \ldots\,,\end{aligned}$$

along with other terms that again provide the gauge-invariant completion. Evaluating at the minimum (11) the kinetic term in (9), we obtain the mass for the trace-$U(1)$ gauge boson A_μ^0, $M_{A^0}^2 = -2\frac{n^2 g^2 m^2}{\lambda}$. This is how the spontaneous symmetry breaking $\mathcal{U}(n) \to SU(n)$ occurs. This can be straightforwardly generalized to the symmetry breaking $\mathcal{U}(n) \times \mathcal{U}(m) \to SU(n) \times SU(m)$. In this case, we need a composite Higgsac field that carries charge n coupled to trace-$U(1)$ of $\mathcal{U}(n)$ and charge $-m$ coupled to trace-$U(1)$ of $\mathcal{U}(m)$, i.e.

$$\begin{aligned}\Phi(x)_{\mathcal{U}(n)\times\mathcal{U}(m)} &= \frac{1}{n!m!}\epsilon_{i_1 i_2,\ldots,i_n}\epsilon^{l_1 l_2,\ldots,l_m}\left(W_{\mathcal{U}(n)}\right)^{i_1}_{j_1} \\ &\quad * \left(W_{\mathcal{U}(n)}\right)^{i_2}_{j_2} * \ldots * \left(W_{\mathcal{U}(n)}\right)^{i_n}_{j_n} \\ &\quad * \phi(x)^{[j_1 j_2 \ldots j_n]}_{[k_1 k_2 \ldots k_m]} * \left(W^{-1}_{\mathcal{U}(m)}\right)^{k_1}_{l_1} \\ &\quad * \left(W^{-1}_{\mathcal{U}(m)}\right)^{k_2}_{l_2} * \ldots * \left(W^{-1}_{\mathcal{U}(m)}\right)^{k_m}_{l_m}\,.\end{aligned} \tag{12}$$

3 Noncommutative standard model

Let us go back now to the model of [7]. Recall that the 'minimal' gauge group for the NC standard model is $G_{\text{NCSM}} = \mathcal{U}(3) \times \mathcal{U}(2) \times \mathcal{U}(1)$. We slightly modify the matter content, however. Usually quarks and leptons are placed in the following G_{NCSM} multiplets:

$$\begin{aligned}L &= \begin{pmatrix}\nu \\ e^-\end{pmatrix}_{\mathrm{L}} \sim (1, 2, 0); \quad E = e^c_{\mathrm{L}} \sim (1, 1, -1)\,, \\ Q &= \begin{pmatrix}u \\ d\end{pmatrix}_{\mathrm{L}} \sim (3, \overline{2}, 0); \quad U = u^c_{\mathrm{L}} \sim (\overline{3}, 1, +1)\,, \\ &\qquad\qquad D = d^c_{\mathrm{L}} \sim (\overline{3}, 1, 0)\,,\end{aligned} \tag{13}$$

and similarly for the remaining generations. The operator of the ordinary weak hypercharge is a superposition of the trace-$U(1)$ generators of G_{NCSM}:

$$Y = -\frac{2}{3}T^0_{\mathcal{U}(3)} - T^0_{\mathcal{U}(2)} - 2T^0_{\mathcal{U}(1)}\,.$$

It is easy to see that Y correctly reproduces the hypercharges (and hence the electric charges) of ordinary quarks and leptons when acting on (13). The above fermionic content is anomalous, however. To cancel the anomalies it is sufficient to introduce a pair of $\mathcal{U}(2)$-doublet lepton fields per generation:

$$L' = \begin{pmatrix} E^+ \\ N' \end{pmatrix}_{\mathrm{L}} \sim (1,2,-1) \text{ and } L'' = \begin{pmatrix} N'' \\ E^- \end{pmatrix}_{\mathrm{L}} \sim (1,2,0)\,. \tag{14}$$

Remarkably, they are vector-like under the G_{SM} subgroup of G_{NCSM}. That means that upon the G_{NCSM} symmetry breaking down to G_{SM}, these extra states might pick up the masses and decouple from the low energy spectrum. The relevant Yukawa interactions can be written using the Wilson lines again:

$$\left(W_{\mathcal{U}(2)} * L' * W^{-1}_{\mathcal{U}(1)}\right)^T * \left(W_{\mathcal{U}(2)} * L''\right) * \Phi_{\mathcal{U}(2)\times\mathcal{U}(1)} + \text{h.c.}\,, \tag{15}$$

where $\Phi_{\mathcal{U}(2)\times\mathcal{U}(1)}$ is the $\mathcal{U}(2)\times\mathcal{U}(1)$ composite Higgsac field analogous to (12):

$$\Phi_{\mathcal{U}(2)\times\mathcal{U}(1)} = \frac{1}{2}\epsilon^{j_1 j_2} W_{\mathcal{U}(1)} * \phi_{[i_1 i_2]} * \left(W^{-1}_{\mathcal{U}(2)}\right)^{i_1}_{j_1} * \left(W^{-1}_{\mathcal{U}(2)}\right)^{i_2}_{j_2}\,, \tag{16}$$

and the proper contraction of gauge indices is understood. The vacuum expectation value of this field, $\langle\Phi_{\mathcal{U}(2)\times\mathcal{U}(1)}\rangle$, provides spontaneous symmetry breaking: $\mathcal{U}(2)\times\mathcal{U}(1) \to SU(2)\times U(1)_{1-2}$, where the surviving $U(1)_{1-2}$ is a linear combination of trace-$U(1)$ of $\mathcal{U}(2)$ and $\mathcal{U}(1)$. At the same time, the pair of left-handed leptons acquires a Dirac mass of the order of $\langle\Phi_{\mathcal{U}(2)\times\mathcal{U}(1)}\rangle$. To break G_{NCSM} down to G_{SM} fully, we must introduce one more Higgsac field, either $\Phi_{\mathcal{U}(3)\times\mathcal{U}(1)}$ or $\Phi_{\mathcal{U}(3)\times\mathcal{U}(2)}$. Upon the condensation of these fields, the only $U(1)$ that remains massless is the usual weak hypercharge field.

4 Discussion and conclusions

We have proposed a mechanism for the spontaneous reduction of the noncommutative gauge symmetry, i.e. $\mathcal{U}(n) \to SU(n)$. This has been achieved through the condensation of composite Higgsac fields (5) and (12) in the trace-$U(1)$ noninvariant, but $SU(n)$ preserving direction. An essential part of our construction was the half-infinite Wilson lines that provide gauge-invariant completion of the Higgsac mechanism proposed earlier in [7].

The proposed mechanism offers new perspectives in realistic model building based on the Weyl–Moyal NC gauge theories. In particular, we have briefly discussed the NC standard model. Besides the spontaneous reduction of $G_{\mathrm{NCSM}} = \mathcal{U}(3)\times\mathcal{U}(2)\times\mathcal{U}(1)$ down to $G_{\mathrm{SM}} = SU(3)\times SU(2)\times U(1)$, with $U(1)$ being the usual weak hypercharge, we have demonstrated that anomalies can be cancelled by the introduction of lepton pairs (per generation of ordinary quarks and leptons) that are vector-like under the G_{SM} (not G_{NCSM}) gauge group. Moreover, the same Higgsac field that provides the breaking $G_{\mathrm{NCSM}} \to G_{\mathrm{SM}}$, couples in a gauge-invariant way to the extra lepton pairs and provides their masses. Thus, the low energy theory can be fully reduced to the standard model with usual spectrum of ordinary quarks and leptons.

The supersymmetric version of the NC standard model proposed in this paper is a subject of interest on its own, which would remove automatically the IR quadratic divergences arising from the UV/IR mixing [18, 19].

One can also construct grand unified models where certain features of the NC standard model discussed here come out naturally. One is the NC trinification model based on the gauge group $\mathcal{U}(3)\times\mathcal{U}(3)\times\mathcal{U}(3)$. Remarkably, the standard minimal fermionic content of the commutative trinification [20] is automatically anomaly-free in the noncommutative case as well. For the spontaneous reduction $\mathcal{U}(3)\times\mathcal{U}(3)\times\mathcal{U}(3) \to SU(3)\times SU(3)\times SU(3)$, one can use the gauge-invariant Higgsac mechanism proposed in this paper.

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